APPLICATION OF A STEP-BY-STEP METHOD TO THE ANALYSIS OF STABILITY OF A COMPRESSED BAR

(PRIMENENIE SHAGOVOGO METODA K ANALIZU USTOICHIVOSTI SZHATOGO STERZHNIA)

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We are proposing a method of analysis of deformable systems based on looking upon the deformation as a process. Time is regarded as a parameter which determines the development of the deformation. The integration with respect to time is carried out on a high-speed electronic digital computer. The boundary value part of the problem is solved by the Galerkin method.

In this approach there is no need to solve nonlinear equations. The values of the parameters from the preceding step are substituted into them. This procedure permits a considerable increase in the number of parameters which are being varied and does not make the solution of nonlinear problems any more complicated than that of linear ones.

As an example, we consider the problem of stability of a bar beyond the elastic limit.

1. A method of analysis of deformable systems called the variational step method, or simply the step method, is proposed in [1].

The deformation of a system is looked upon as a process, regardless of the fast or slow rate at which the external forces may be changing. Time is introduced as an independent variable and the following equations of motion are formed:

$$
\begin{aligned}
-\rho\frac{\partial^2 u}{\partial t^2} + \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} &= 0 \\
-\rho\frac{\partial^2 v}{\partial t^2} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} &= 0 \qquad (1.1)
\end{aligned}
$$

$$- \rho \frac{\partial^2 w}{\partial t^2} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

Stresses are given as functions of deformations by the relations of elasticity or plasticity which we symbolically write down in the form

$$\sigma = \sigma (\varepsilon,) \tag{1.2}$$

Deformations, in turn, are determined by the relations

$$\varepsilon = \varepsilon (u, v, w) \tag{1.3}$$

which, generally speaking, are nonlinear.

We approximate the displacements u, v and w by the functions which satisfy the boundary conditions

$$\begin{aligned} u &= A_1 u_1 (x, y, z) + A_2 u_2 (x, y, z) + \ldots + A_m u_m (x, y, z) \\ v &= B_1 v_1 (x, y, z) + B_2 v_2 (x, y, z) + \ldots + B_m v_m (x, y, z) \\ w &= C_1 w_1 (x, y, z) + C_2 w_2 (x, y, z) + \ldots + C_m w_m (x, y, z) \end{aligned} \tag{1.4}$$

where A_i, B_i and C_i are time-dependent parameters.

Substituting u, v and w into equations (1.1) to (1.3) and using the Galerkin method we obtain systems of ordinary differential equations

$$\begin{aligned} &\ddot{A}_1 + a_{12}\ddot{A}_2 + a_{13}\ddot{A}_3 + \ldots + L_1 (A_i, B_i, C_i) = 0 \\ &a_{21}\ddot{A}_1 + \ddot{A}_2 + a_{23}\ddot{A}_3 + \ldots + L_2 (A_i, B_i, C_i) = 0 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ddot{B}_1 + b_{12}\ddot{B}_2 + b_{13}\ddot{B}_3 + \ldots + M_1 (A_i, B_i, C_i) = 0 \\ &b_{21}\ddot{B}_1 + \ddot{B}_2 + b_{23}\ddot{B}_3 + \ldots + M_2 (A_i, B_i, C_i) = 0 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ddot{C}_1 + c_{12}\ddot{C}_2 + c_{13}\ddot{C}_3 + \ldots + N_1 (A_i, B_i, C_i) = 0 \\ &c_{21}\ddot{C}_1 + \ddot{C}_2 + c_{23}\ddot{C}_3 + \ldots + N_2 (A_i, B_i, C_i) = 0 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{1.5}$$

If the functions u_i, v_i and w_i are orthogonal, equations (1.5) are simplified and take the form

$$\begin{aligned} &\ddot{A}_1 + L_1 (A_i, B_i, C_i) = 0, \qquad \ddot{A}_2 + L_2 (A_i, B_i, C_i) = 0, \ldots \\ &\ddot{B}_1 + M_1 (A_i, B_i, C_i) = 0, \qquad \ddot{B}_2 + M_2 (A_i, B_i, C_i) = 0, \ldots \\ &\ddot{C}_1 + N_1 (A_i, B_i, C_i) = 0, \qquad \ddot{C}_2 + N_2 (A_i, B_i, C_i) = 0, \ldots \end{aligned} \tag{1.6}$$

Here L_i, M_i and N_i are quantities which depend on the parameters A_i, B_i and C_i.

System (1.6) is integrated step-by-step on a high-speed electronic digital computer. The quantities L_i, M_i and N_i are expressed in terms of A_i, B_i and C_i of the preceding step by a successive transition from equations (1.4) to equations (1.3), (1.2) and (1.6). As a result of integration we obtain displacements, stresses and deformations as functions of time. It is assumed that the functional dependence of forces upon time is given. In particular, if a case of static loading is considered it can be assumed that forces change proportionally to time and a sufficiently small coefficient of proportionality should be chosen.

Rapid variations of forces or sharp changes in deflections in connection with loss of stability must give rise to damped oscillations in the system. A quantitative estimate of the damping can be obtained by introducing terms which contain the rates of deformation into equations (1.2). For that purpose we have to make use of one of the hypotheses concerning the properties of the material. The expressions for L_i, M_i and N_i will then contain not only the parameters A_i, B_i and C_i, but also their first derivatives.

If damping is not taken into consideration the picture of deformation may be distorted by undamped oscillations which hamper the analysis of the computation results. Consequently, for the sake of clarity it is sometimes expedient deliberately to introduce linear damping directly into equations (1.6) in the following form:

$$\begin{array}{ll} \ddot{A}_1 + \alpha_1\dot{A}_1 + L_1 = 0, & \ddot{A}_2 + \alpha_2\dot{A}_2 + L_2 = 0, \ldots \\ \ddot{B}_1 + \beta_1\dot{B}_1 + M_1 = 0, & \ddot{B}_2 + \beta_2\dot{B}_2 + M_2 = 0, \ldots \\ \ddot{C}_1 + \gamma_1\dot{C}_1 + N_1 = 0, & \ddot{C}_2 + \gamma_2\dot{C}_2 + N_2 = 0, \ldots \end{array} \tag{1.7}$$

The coefficients α_1, α_2, ..., β_1, β_2, ..., γ_1, γ_2, ... must not be chosen too large, so that the motion retains the oscillatory character, and not too small, so that the damping is sufficiently noticeable.

The simplest way to resolve this question is to linearize the expressions for L_1, L_2, ..., M_1, ... and consider equations (1.7) as independent.

If loading is taking place at a slow rate, one can neglect the inertia forces, discard the second derivatives of the parameters A_1, A_2, ..., B_1, ... and consider the system to be viscously deformable. Then it is only necessary to coordinate the increment of the integration step with the values of the coefficients α_1, α_2, ..., β_1, β_2, ..., γ_1, γ_2, ...

Generally speaking, the increment Δt of the integration step must in any case be essentially smaller than the period of the natural oscillations corresponding to the highest partial frequency. This, by the way, imposes a limitation on the number of parameters which may be varied. When their number is large the highest partial frequency increases considerably, so that a very small increment of the time step must be chosen. As a result, the computer time is increased.

If we decide not to consider the process as taking place in time and instead go into the usual analysis of the forms of equilibrium, then instead of the differential equations (1.7) we obtain the following system

$$\begin{aligned} &L_1(A_i, B_i, C_i) = 0, \qquad L_2(A_i, B_i, C_i) = 0, \ldots \\ &M_1(A_i, B_i, C_i) = 0, \qquad M_2(A_i, B_i, C_i) = 0, \ldots \\ &N_1(A_i, B_i, C_i) = 0, \qquad N_2(A_i, B_i, C_i) = 0, \ldots \end{aligned} \tag{1.8}$$

It has to be solved for the parameters A_i, B_i and C_i which are being varied. As a rule, for nonlinear relations and a large number of parameters this cannot be accomplished. However, in the step-by-step method one simply has to substitute the values of A_i, B_i, C_i of the preceding step into the expressions for L_1, L_2, ... which is always easily done.

The merits of the step-by-step method lie in the fact that in it the borderline between the linear and nonlinear systems, the small and large displacements, the statics and dynamics, is completely wiped away and there is almost no difference between the problems of elasticity and plasticity. A new possibility of solving problems in creep and plasticity which are related to the history of loading thus presents itself. Unlike the standard methods, here the number of parameters being varied can be increased considerably. To that end all the operations of substitution in the transition from equations (1.3) to equations (1.2) and further, to equations (1.1), must be performed on a computer.

2. Let us consider the problem of stability of a bar (Fig. 1) compressed beyond the limit of elastic deformations.

This problem, as any other related to stability of plastically deformable systems, first of all needs a formulation of underlying principles on which the analysis is based. It is generally understood that stability of a system is a property of returning to its initial state after the causes which have produced relatively small disturbances are removed. It can be stated in advance that whenever plastic deformations are present the system, generally speaking, does not possess that property.

If in a straight axially compressed bar there arise no plastic deformations, then one can always select a sufficiently small deflection such that the resulting sum of bending and compression stresses will not exceed the elastic limit. At the same time, a somewhat larger deflection may cause residual deformations, although the axial force remains the same, so that the bar will not return into its initial state. If a straight bar is compressed beyond the elastic limit then for any deflection, no matter how small, it will not regain its original shape when released.

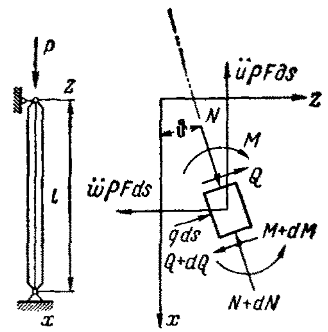

Fig. 1.

It is clear that the problem of stability of a plastically deformable system requires a special approach. Here the decisive factor becomes the history of loading and history of tests performed (deviations of the system from the initial equilibrium configuration). The correct approach here, and it seems the only correct approach, is to consider the loss of stability of a plastically deformable system not as a set of possible forms of equilibrium, but as a process. The step-by-step method permits an investigation of this kind to be carried out to completion.

Thus, we have a two-hinged bar (Fig. 1).

The vertical and horizontal components of the internal forces in a cross-section of the bar (Fig. 1) are

$$X = N\cos\vartheta - Q\sin\vartheta, \quad Y = N\sin\vartheta + Q\cos\vartheta$$

where ϑ is the angle of rotation of the section. We do not impose any limitations on the angle ϑ.

Then we form the equations of motion

$$\rho F\frac{\partial^2 u}{\partial t^2} + \frac{\partial}{\partial s}(N\cos\vartheta - Q\sin\vartheta) = -q\sin\vartheta$$

$$\rho F\frac{\partial^2 w}{\partial t^2} + \frac{\partial}{\partial s}(N\sin\vartheta + Q\cos\vartheta) = q\cos\vartheta$$

$$(Q = \partial M/\partial s) \qquad (2.1)$$

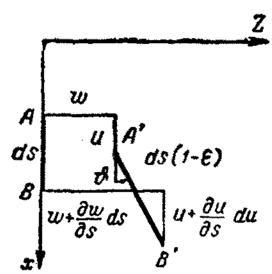

Fig. 2.

Let us consider the closed polygon $AA'B'B$ (Fig. 2). Setting the sums of projections of the segments on the z- and x-axes equal to zero we obtain

$$(1-\varepsilon)\sin\vartheta - \frac{\partial w}{\partial s} = 0, \qquad (1-\varepsilon)\cos\vartheta - 1 - \frac{\partial u}{\partial s} = 0$$

Hence

$$\sin\vartheta = \frac{1}{R}\frac{\partial w}{\partial s}, \qquad \cos\vartheta = \frac{1}{R}\left(1 + \frac{\partial u}{\partial s}\right), \qquad \varepsilon = 1 - R$$
$$R = \sqrt{\left(1 + \frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial w}{\partial s}\right)^2} \tag{2.2}$$

For the curvature $\kappa = \partial\vartheta/\partial s$ we obtain

$$\varkappa = \frac{\cos\vartheta}{R}\frac{\partial^2 w}{\partial s^2} + \frac{\sin\vartheta}{R}\frac{\partial^2 u}{\partial s^2} \tag{2.3}$$

The stresses are determined in terms of κ and ε by means of the compression diagram, and through stresses we obtain forces and moments

$$N = \int_F \sigma dF, \qquad M = \int_F \sigma z dF \tag{2.4}$$

Let us assume that the cross-section has the form of a rectangle with the shorter side h and let us rewrite the equations in the dimensionless form. Thus

$$u = hu_0, \quad w = hw_0, \quad s = l\xi, \quad N = EFN_0, \quad M = EFlM_0$$
$$Q = EFM_0', \quad t = \tau\sqrt{\frac{lh\rho}{E}}, \quad q = \frac{q_0 EF}{l}, \quad \frac{\partial(\ )}{\partial\xi} = (\ )', \quad \frac{\partial(\ )}{\partial\tau} = (\dot{\ })$$

Equations (2.1) through (2.3) then become

$$\ddot{u}_0 + (N_0\cos\vartheta - M_0'\sin\vartheta)' = -q_0\sin\vartheta$$
$$\ddot{w}_0 + (N_0\sin\vartheta + M_0'\cos\vartheta)' = q_0\cos\vartheta \tag{2.5}$$

$$\sin\vartheta = \frac{h}{l}\frac{w_0'}{R_0}, \qquad \cos\vartheta = \frac{1}{R_0}\left(1 + \frac{h}{l}u_0'\right), \qquad \varepsilon = 1 - R_0 \tag{2.6}$$

$$R_0 = \sqrt{\left(1 + \frac{h}{l}u_0'\right)^2 + \left(\frac{h}{l}w_0'\right)^2} \tag{2.7}$$

$$\varkappa h = \frac{h}{l}\vartheta' = \frac{1}{R_0}\left[\frac{h^2}{l^2}w_0''\cos\vartheta - \frac{h^2}{l^2}u_0''\sin\vartheta\right] \tag{2.8}$$

Now we introduce the designation $\eta = z/h$; for N_0 and M_0 we obtain

$$N_0 = \int_{-1/2}^{+1/2}\frac{\sigma}{E}d\eta, \qquad M_0 = \frac{h}{l}\int_{-1/2}^{+1/2}\frac{\sigma}{E}\eta d\eta \tag{2.9}$$

We assume that

$$w_0 = \sum_{1,2,3\ldots} A_n\sin\pi n\xi, \qquad u_0 = B_0(1-\xi) + \sum_{1,2,3\ldots} B_n\sin\pi n\xi \tag{2.10}$$

where A_n and B_n depend on dimensionless time.

Let us substitute u_0 and w_0 into equations (2.7) and (2.8). Then from the compression diagram find σ and by integration of (2.9) determine N_0 and M_0 for a number of cross-sections.

Now substitute N_0, M_0, u_0 and w_0 into the equations of motion (2.5) We use the Galerkin method. Multiplying both equations by $\sin \pi i\xi$ and integrating from 0 to 1, we obtain

$$\int_0^1 \left[\ddot{B}_0 (1-\xi) + \sum \ddot{B}_n \sin \pi n\xi\right] \sin \pi i\xi d\xi +$$

$$+ \int_0^1 (N_0 \cos \vartheta - M_0' \sin \vartheta)' \sin \pi i\xi d\xi = -\int_0^1 q_0 \sin \vartheta \sin \pi i\xi d\xi$$

$$\int_0^1 \sum A_n \sin \pi n\xi \sin \pi i\xi d\xi +$$

$$+ \int_0^1 (N_0 \sin \vartheta + M_0' \cos \vartheta)' \sin \pi i\xi d\xi = \int_0^1 q_0 \cos \vartheta \sin \pi i\xi d\xi$$

In order to avoid differentiating the functions N_0 and M_0 which are determined for a number of cross-sections, we take the second integral by parts twice.

As a result we obtain

$$\ddot{A}_i + \alpha_i \dot{A}_i - \Phi_i = 0, \qquad \ddot{B}_i + \beta_i \dot{B}_i - F_i = 0 \tag{2.11}$$

where

$$\Phi_i = 2\int_0^1 \left[(q_0 + \pi^2 i^2 M_0) \cos \vartheta \sin \pi i\xi + \pi i \left(N_0 + M_0 \frac{l}{h} h\varkappa\right) \sin \vartheta \cos \pi i\xi\right] d\xi$$

$$F_i = 2\left\{\int_0^1 \left[-(q_0 + \pi^2 i^2 M_0) \sin \vartheta \sin \pi i\xi + \right.\right.$$
$$\left.\left. + \pi i \left(N_0 + M_0 \frac{l}{h} h\varkappa\right) \cos \vartheta \cos \pi i\xi\right] d\xi - \frac{\pi}{i} \ddot{B}_0 - \frac{\pi}{i} \beta_i \dot{B}_0\right\} \tag{2.12}$$

A linear damping with arbitrary parameters α_i and β_i has been introduced into expressions (2.11). The index i takes the values 1, 2, ..., m, where m is the index of the last term in expansions (2.10). The equations are connected through the quantities Φ_i and F_i which depend on $A_1, A_2, \ldots, B_1, B_2, \ldots$.

We will assume that the bar is compressed in a rigid hydraulic press and that its ends are brought together according to a given law

$$B_0 = f(\tau)$$

The axial force on the upper end of the bar obviously equals

$$P = \left| N \cos\vartheta - \frac{\partial M}{\partial s} \sin\vartheta \right|_{s=0}$$

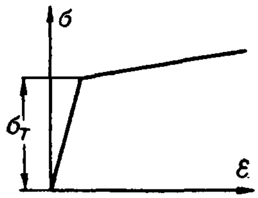

Fig. 3.

or in dimensionless form

$$P_0 = \frac{P}{EF} = | N_0 \cos\vartheta - M_0' \sin\vartheta |_{\xi=0} \qquad (2.13)$$

As the end sections are brought together, the force P_0 first increases and then, when noticeable transverse displacements are formed, it decreases. As is usually done in testing, we will take the maximum value of the force to be the index of stability. This force can be called critical, not in the sense of bifurcation of the forms of equilibrium but rather in the sense of carrying capacity, which essentially is what we require.

In order to make the following computations more concrete let us take the compression diagram which is schematically given in the form of two straight lines (Fig. 3):

$$\sigma = E\varepsilon, \qquad \sigma - \sigma_T = D\left(\varepsilon - \frac{\sigma_T}{E}\right)$$

and let us introduce the dimensionless parameters

$$a = \frac{\sigma_T}{E}, \qquad b = \frac{D}{E}$$

Then the equations of the sections of diagram become

$$\frac{\sigma}{E} = \varepsilon, \qquad \frac{\sigma}{E} = a(1-b) + b\varepsilon$$

We choose the quantities a and b and the ratio h/l. Then we introduce the initial values of the coefficients A_i and B_i. The axial displacement u is assumed to be initially equal to zero. Therefore $B_1 = B_2 = \ldots = 0$.

The initial deflection w_0 is given by the parameters A_{10}, A_{20}, ... In the computations we have considered only the influence of A_{10} and A_{20}, i.e. we have assumed that the bar could be initially bent in the shape of one or two half-waves. The initial values of the following terms of the expansion were in all cases assumed equal to zero.

Altogether 16 varying parameters have been introduced into the computation

$$B_1, \quad B_2, \ldots, B_8, \quad A_1, \quad A_2, \ldots, A_8$$

We divide the bar into several portions and for each section from formulas (2.10) we compute the values of w', u', w'', u'', $h\kappa$, $\sin\vartheta$, $\cos\vartheta$ and ε with the initial values of A_{10} and A_{20}.

As the above operation is carried out the first time, the obtained value of the initial curvature is recorded and in the subsequent steps is subtracted from the new values of $h\kappa$ each time. This is necessary since the bending moment is determined by the difference in curvature (the new and the initial one).

Then the deformation at a number of points in each section is computed

$$\varepsilon_z = \varepsilon + h\varkappa\,\frac{z}{h} = \varepsilon + h\varkappa\eta$$

Each section was vertically divided into 10 layers and in each layer the value of ε_z was computed and stored in the memory core, to be used in the next time step.

Using the value of ε_z we find the stresses from the compression diagram. In doing it the computer program uses a logical comparison with the preceding step, so that the conditions of loading and unloading can be distinguished.

Integrating (2.9), we obtain the values of N_0 and M_0 for each cross-section.

We substitute N_0 and M_0 into expressions (2.12) and for $i = 1, 2, 3 \ldots 8$ we carry out the integration with respect to ξ. Then from equations (2.11) we find the increments $\Delta\dot{A}_i$, $\Delta\dot{B}_i$, ΔA_i, ΔB_i. These increments are added to the preceding values of $\dot{A}_i$, $\dot{B}_i$, A_i and B_i and the cycle is repeated.

It was assumed in the computations that the end sections are brought together at a constant rate and hence

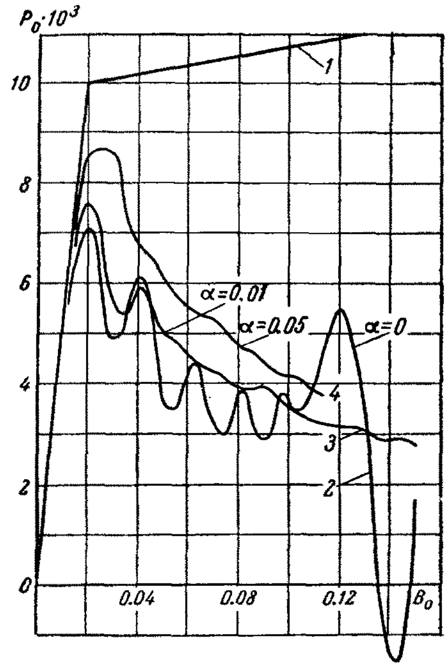

Fig. 4.

$$B_0 = K\tau \tag{2.14}$$

The force P is determined from relation (2.13). The results of some computations are considered below.

Figure 4 shows the plot of the force P_0 versus the relative shortening of the distance between the ends of the bar B_0 with the following parameters:

$$\frac{h}{l} = 0.05, \qquad a = \frac{\sigma_T}{E} = 0.001, \qquad b = \frac{D}{E} = 0.015$$

It is assumed that the bar has the initial deflection of the form of a sinusoidal half-wave with the relative amplitude

$$A_{10} = w_{\max} / h = 0.1$$

The value of K in expression (2.14) is assumed to be 10^{-4}.

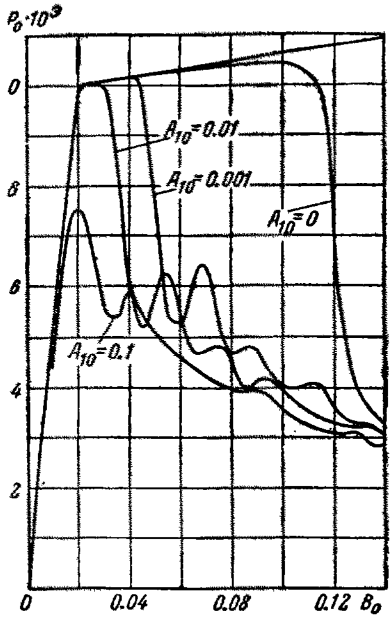

Fig. 5.

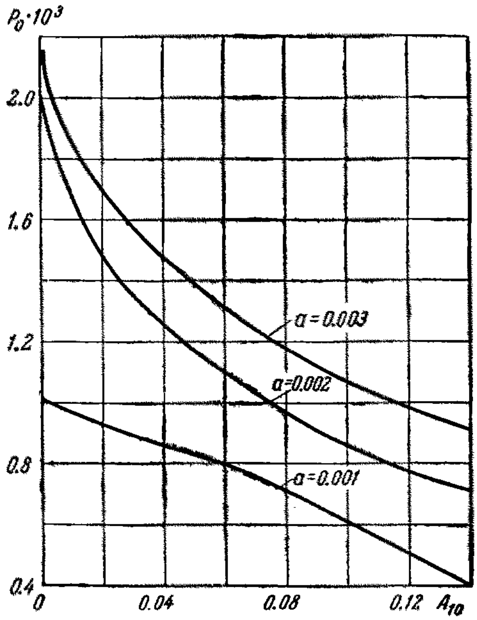

Fig. 6.

With such a rate of shortening of the distance between the end sections, the plastic deformations of a straight bar appear approximately in two periods of the natural transverse oscillations of the fundamental tone.

Number 1 in Fig. 4 designates the compression diagram in the coordinates B_0, P_0. Curves 2, 3 and 4 differ by the value of the arbitrarily introduced coefficients of linear damping α_i. For the rather high rate of loading assumed above, the influence of the coefficient of damping upon the value of the critical force is quite pronounced. If we take $\alpha_i = 0$ (Curve 2), the picture of the decrease of loading is very much distorted by undamped oscillations. In the further plots of $P_0 = f(B_0)$ the assumption was $\alpha_i = 0.01$. In the investigation of the functional dependence of P_{max} upon various parameters, the linear damping was assumed equal to zero so as to obtain a lower value for P_{max}.

Figure 5 shows the variation of the shape of Curve $P_0 = f(B_0)$ for different values of the initial deflection A_{10} of the same bar. Figure 6 shows the dependence of P_{max} on A_{10}.

Here, in determining P_{max}, the rate of loading was 10 times lower than that with which the curves in Fig. 5 were constructed, i.e. we had $K = 10^{-5}$.

It is evident from the curves shown that the initial deflection is the determining factor in the carrying capacity of a bar.

In principle, the question of what the computer would give if no initial deflection of the bar is introduced, is of some importance.

It has become evident that the perturbations of the algorithm (step-by-step variation of the parameters, rounding off of numbers) have turned out to be sufficient to have the loss of carrying capacity recorded at a certain value of the shortening of distance between the end sections. This can be observed, for instance, from the curves of Fig. 5.

Figure 7 shows the dependence of P_{max} on $a = \sigma_T/E$ for several values of h/l. As should be expected, for every value of h/l there exists a value of σ_T beyond which P_{max} does not change with increasing σ_T, insofar as the loss of carrying capacity takes place within the elastic region.

It is essential to point out that even with the very low rate of loading selected ($K = 10^{-5}$), the loss of carrying capacity occurred with a force considerably higher than the Euler's force.

For a longer bar this discrepancy, naturally, becomes more pronounced, since the period of natural oscillations is increased and the rate of loading remains unchanged.

The influence of the loading rate is illustrated by the curves shown in Fig. 8. In both cases of loading the bar has the initial deflection $A_{10} = 0.1$ and the ratio $h/l = 0.05$. The rate at which the end sections are brought together in one bar is 10 times higher than in the other. As a result P_{max} has increased considerably and, due to the longitudinal inertia forces, the loading curve has risen above the values given by the compression diagram.

A comparison of loading curves was conducted for two identical bars with different shapes of the initial deflection curves. One bar was initially bent in the shape of a sinusoidal half-wave, the other in two half-waves. The amplitudes of the deflections were the same. The value of P_{max} for the bar bent in two half-waves turned out to be substantially higher than for the bar bent in one half-wave.

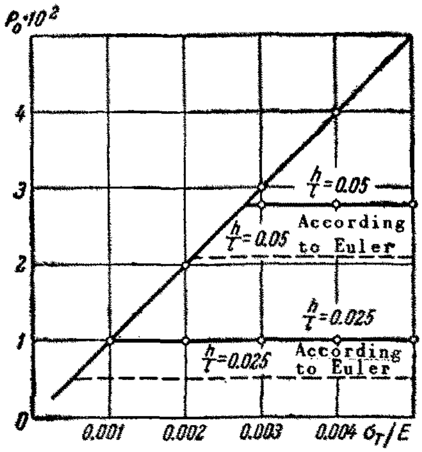

Fig. 7.

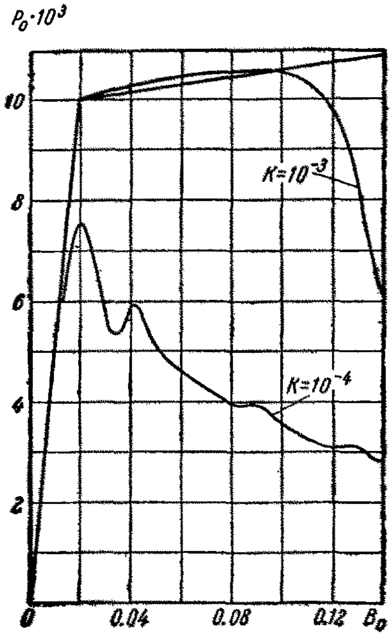

Fig. 8.

As the deflections increased, the highest amplitude in both cases was attained by the form of bending in one half-wave, which should be expected.

For further investigation, the problems of dynamics of a plastically deformable bar are of utmost interest.

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